Quanser Qube 2 – Report

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# Introduction

In the following chapters we’ll resume our work on the project of the Inverter Rotary Pendulum Quansar Cube 2 by explaining how did we approach to it, starting from the mechanical equations, to a state space model used in the simulations. An important aspect was the parameter estimation, including Frictions, Disturbances and others.

Once an good model was found we could go trough different control scheme, based on output feedback and state feedback.

# Equations of motion

In this section we will compute the equations of motion of the system, starting from the mechanical part and then adding the DC motor effects.

## Lagrange Equations

A simplified representation of the QUBE-Servo 2 is shown below, along with some necessary notation :

* Θ : Motor angle (Positive clockwise)
* Φ : Pendulum angle (Positive counterclockwise and null for the pendulum pointing
* Jr : Rod moment of inertia with respect to its rotation axis
* Lr : Rod Lenght
* Jp : Pendulum moment of inertia with respect to its centre of mass
* Lp : Pendulum lenght
* lp : Distance of the pendulum centre of mass from the rotating axis
* mp : Pendulum mass

IMAGE

With “rod” we indicate the element of the QUBE-Servo rigidly connected to the motor shaft that holds the pendulum. The Position of the origin of the fixed reference frame and its axes orientation are shown in the drawing above.

According to Lagrange method, the equations of motion of the system can be written as (1.1.1):

(1.1.1)

Where represents our state space variables che sono , sapendo ora che l’energia delle forze smorzanti può essere omessa Possiamo riscrivere la formula di lagrange dopo le semplificazioni e scrivendo un’equazione per ogni stato e considerando come differenza tra energia cinetica e potenziale, vedi equazione (1.1.2)

(1.1.2)

Dove l’energia cinetica complessiva sarà data dalla somma di energia cinetica del pendolo sommata all’energia cinetica del rod (1.1.3)

(1.1.3)

dove rappresenta la velocità del centro di massa del pendolo, dove i suoi contributi sui 3 assi possono essere espressi come descritto in (1.1.4)

(1.1.4)

dove ogni componente di velocità può essere descritto in base alla derivate delle coordinate del centro di massa del pendolo (1.1.5)

(1.1.5)

che si possono trovare esplicitamente calcolate nel capitolo 1.2 Reference System.

Per quanto riguarda l’energia potenziale del sistema sarà solo il contributo del pendolo (1.1.6)

(1.1.6)

Tenendo ora in considerazione che controlleremo una coppia che è posta su un motore direttamente collegato al rod e con Nessun lavoro esterno invece sul pendolo, possiamo esprimere (1.1.7)

(1.1.7)

Risolvendo il sistema nell’equazione (1.1.2) possiamo trovare le equazioni che descrivono totalmente la dinamica del nostro sistema (1.1.8)

(1.1.8)

Which can be also expressed as equation (1.1.4)

(1.1.4)

With the following matrices (1.1.5)

## Reference System

Metti tante immagini del sistema di riferimento in the equation (1.2.1)

(1.2.1)

## DC Motor

Torque is generated by the DC permanent magnets motor according to the following electrical dynamic equation (1.3.1)

(1.3.1)

Following the necessary notation :

* : armature voltage
* : armature current
* : armature resistance
* : armature inductance
* E : back electromotive force
* : back-emf constant
* : torque constant (it holds )

As it will be clear in the next section where the parameter values are introduced, the armature inductance is small enough so that the term can be reasonably neglected (in other words, the motor electrical time constant is small).

By doing so we are neglecting the DC motor electrical dynamics, and as a consequence the electrical torque can be expressed as in equation 1.3.2 :

(1.3.2)

There is no need to introduce the motor mechanical dynamic equation as the motor is directly coupled to the rod without transmission gears and the motor moment of inertia is already accounted inside , the rod moment of inertia (which, as already stated, is considered with respect to the motor rotating axis).

## Final Mechanical Equations

By summarizing the entire chapters 1.1, 1.2, 1.3 and 1.4, we can reach to a final equations of the systems,

# Parameters Estimation

## Static Friction

## Dynamic Friction

## Cable Disturbance

## Motor Parameters

## Inertia

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## System Parameters

## Conclusions

# Controllers and state estimation

## Output Feedback Controller (PI)

## State Feedback Controllers (PP-LQR)

## State Observer (KF-PP)

## Swing-up Controller (PP)

## Swing-up Controller (Energy Based)

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